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THESIS

**A SCHEDULING MODEL FOR
THE U.S. MARINE CORPS COMMUNICATION-
ELECTRONICS SCHOOL**

by

Barry D. Justice

September 1993

Thesis Co-Advisors:

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**A SCHEDULING MODEL FOR THE U.S. MARINE CORPS
COMMUNICATION-ELECTRONICS SCHOOL**

by

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Captain, United States Marine Corps
B.S. EE, Purdue University, 1984

Submitted in partial fulfillment
of the requirements for the degree of

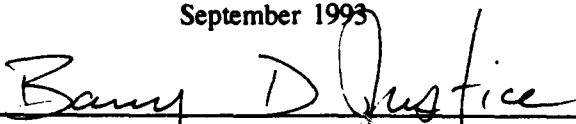
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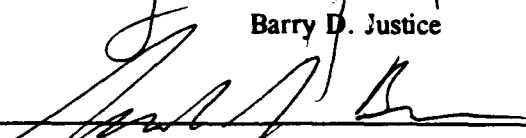
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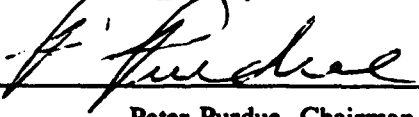

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ABSTRACT

This thesis presents a mixed integer program (MIP) to schedule sequences of classes attended by Marines at the Marine Corps Communication-Electronics School in order to qualify them for communications and electronics military occupational specialties. The schedule determines the starting dates and the number of students to enroll in each instance or "class" of various course types. The courses follow a specific sequence and many classes of a course may be scheduled within a fiscal year. Students attend one or more of the courses and may wait some time for a class of a subsequent course to convene. The objective of the MIP is to reduce the amount of delay students incur while waiting for classes of additional courses to start in the sequence. Due to the size and complexity of the model, the MIP initially schedules classes with a weekly resolution but then adjusts the starting dates to produce a daily schedule. For 1993 data, the MIP is solved in less than 10 minutes on a desktop computer (80486 processor at 66Mhz and at least 64M RAM) and produces a schedule which has 62% less delay than the actual schedule for that year.

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The reader is cautioned that computer programs developed in this research may not have been exercised for all cases of interest. While effort has been made, within the time available, to ensure that the programs are free of computational and logic errors, they cannot be considered validated. Any application of these programs without additional verification is at the risk of the user.

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EXECUTIVE SUMMARY

This thesis presents a method to improve the efficiency of the training of enlisted Marines in communications and electronics Military Occupational Specialties (MOSs). A mixed integer program (MIP) is developed which schedules the courses provided by the Marine Corps Communication-Electronics Schools (MCCES). The objective is to minimize the delay students incur while waiting for follow-on courses.

To meet the annual MCCES training requirements (e.g., 90,804 man-days of class work in 1993), more than one instance of each course is usually required. Each instance of a course is designated a "class" and a completed schedule requires specifying the starting dates and the number of students to be enrolled in each class. The courses follow a specific sequence with students proceeding to subsequent courses after successful completion of the initial course. The number of students who can be scheduled for a class of a course must remain between a maximum and minimum for each course type. The number of concurrent classes of a given course must remain below a maximum for that course and a minimum number of days between starting dates of classes must be maintained.

These constraints may cause students to incur some delay while waiting for classes of a subsequent course to begin. The goal of this thesis is to develop a schedule which

minimizes the student delay so that students may proceed more quickly to their subsequent assignments.

The size of the MIP created by a set of 17 courses and a planning horizon of 236 training days is large and a solution cannot be obtained. When the planning horizon is scaled to a weekly resolution, the size of the MIP is reduced by nearly 80% and solution times become manageable. A rescaling of the planning horizon back to a daily resolution along with a partial correction for delay caused by scaling is then performed. Fractional attrition of students does create difficulty in obtaining schedules having integer class sizes, but this is largely overcome by assuming classes have known integer values for attrition.

Regardless of the method used to schedule classes, limitations in resources will impose some delay of students in completing training. However, an existing 1993 schedule of classes for a set of courses from MCCES exhibits 7,819 man-days of student delay, which adds 8% to the total man-days required for training, while a schedule developed by the MIP using the same 1993 data adds 2,964 man-days of delay which is only 3% of the total. Schedules created by the MIP can be obtained on a desktop computer (80486 microprocessor at 66MHz with at least 64M RAM) in approximately 30 minutes.

I. INTRODUCTION

Every entrant into the armed services must undergo initial training to qualify for service in a specific military occupational specialty (MOS). In the Marine Corps, the entrant must pass a series of one or more courses to acquire an MOS. This thesis presents a mixed integer programming (MIP) model to aid scheduling of courses and students in communications and electronics MOSs. The goal of the model is to reduce the waiting time of the students between subsequent courses so that training is accomplished efficiently and the students move to their subsequent assignments more quickly.

A. BACKGROUND

The Marine Corps Communication Electronic School (MCCES) located at the Marine Corps Air Ground Combat Training Center, Twentynine Palms, California, is responsible for the training of entry level Marines in electronic fundamentals, operational communications, air control/antiair warfare operations and communications-electronics maintenance. MCCES has developed specific courses of instruction designed to meet the requirement for MOS training in these areas.

Headquarters Marine Corps identifies the annual training requirement for each course in terms of total number of students who must **start** training and then directs the school

to develop a schedule for the courses it provides. The schedule must identify the number of students in and the starting date of each occurrence of a course. The number of possible days which may be considered for starting courses are referred to as "training days" and excludes weekends, federal holidays and miscellaneous other days designated by the commander of MCCES.

Each occurrence of a course is called a "class" and is designated by course name, a number identifying which occurrence it is in the fiscal year and by the fiscal year in which the class will be conducted. An example of the class designation for the first basic electronics course (BEC) conducted in fiscal year 1993 would be, BEC 1-93. Typically, more than one class for each course is necessary to satisfy the year's training requirements because the requirements can be large and the class sizes are limited. The enlisted assignment branch at Headquarters Marine Corps uses the schedule produced by MCCES when they identify and order students to receive MOS training after successful completion of other initial training. Both officer and enlisted personnel undergo training at MCCES. However, since the number of officers is only a small percentage of the total number of personnel trained at MCCES, this thesis considers the scheduling of classes for enlisted personnel only.

Much of the MOS training provided by MCCES requires the progression of students through a sequence of courses, one at

a time, beginning with an elementary course followed by a number of specialized courses. Courses in a sequence have a specific ordering which must be followed, establishing a predecessor-successor relationship. For each course (except the first course) in a sequence, a predecessor course is uniquely defined. Therefore, all the courses a student has attended upon completion of training at MCCES may be identified by the last course taken and the unique predecessor property.

Upon completion of a course, a student will be associated with one of three exclusive categories: 1. The student has completed training at MCCES. 2. The student starts a successor course on the next training day after completion of a predecessor course. 3. The student is placed in "inventory" awaiting additional training.

Progression of students through a sequence of courses, one at a time, is a common method of training for many military MOSs, especially those MOSs which are highly technical. Pilot training, electronics maintenance and avionics are examples of the types of MOSs which require completion of sequential courses. Much of the research in this thesis is also applicable to such MOSs.

B. RELATED STUDIES

There has been much written on the subject of scheduling. The problem presented in this thesis has both similarities and

differences with many types of scheduling problems and a discussion of scheduling problems most closely related to the MCCES scheduling problem follows.

MOS training of Marines is somewhat analogous to a frequently seen scheduling problem often referred to as "timetabling". The timetabling problem arises when students and a teacher (or teachers) must meet over a finite period of time, like a semester, and require resources, e.g., rooms, teaching aids, etc. [Ref. 1]. The nature of this problem is that a fixed number of classes of a course type must be assigned to specified blocks of time within the planning period. Timetabling typically does not involve the scheduling of courses which have a predecessor-successor relationship and differs from the MCCES scheduling problem in this regard. Additionally, a student at MCCES attends only one course at a time and there is no need to deconflict two or more courses competing for assignment to a specific time block.

Attendance at one or more courses which have a specified sequence is similar to "job-shop" scheduling. As defined by Gere [Ref. 2], job-shop scheduling specifies a number of jobs, each comprising one or more operations to be performed in a specified sequence on specified machines and requiring certain amounts of time, to be scheduled such that due dates associated with each job will be met, or, failing this, some measure, such as the amount of time exceeding the

due date, is minimized. Davis [Ref. 3] lends insight to this broad definition by observing that, typically, the jobs are assumed to be one-of-a-kind orders, with simple (mostly serial) precedence orderings among operations, and requirements of only one machine for each operation. Also, it is generally assumed that only one machine is available for each operation. The Marine Corps MOS training problem differs from the job-shop scheduling problem in that the number of jobs (students) to be processed is variable between certain limits, may vary from machine to machine (course to course), and there may be more than one machine (course) of each type running at a given time.

Another similar type of scheduling problem is that of the "resource-constrained project scheduling" problem. As presented by Fisher, [Ref. 4], the resource-constrained project scheduling problem requires the determination of a series of start times of specified tasks which minimize some function of the task completion times. The tasks are constrained by predecessor-successor relationships expressed by a network and expend specified amounts of various scarce resources. The resource constrained project scheduling problem addresses many of the desired objectives for project completion including minimizing project duration, minimizing project cost given performance payments and penalties, and minimizing the consumption of critical resources [Ref. 5]. This type of scheduling problem

is related to the MCCES scheduling problem in that resources (instructors) are expended in accomplishing the tasks (courses) and the tasks have a specified predecessor-successor relationship. However, like the job-shop scheduling problem this problem does not address the variability in job (class) sizes from task to task (course to course).

C. PURPOSE AND SCOPE

The purpose of this thesis is to reduce the amount of delay encountered by students waiting for additional courses. The thesis will identify a schedule of classes which specifies the starting dates for classes of each course, along with the number of students which must attend each class.

The model presented in this thesis may be used for the scheduling of any courses provided by MCCES, but the primary focus is on courses which provide entry level training of students in communications and electronics maintenance. The model is not intended to replace the scheduler. The scheduler will still need to apply judgement to adjust the schedule to accommodate changes in requirements, resources, or course-specific data which occur mid-year.

The number of students **finishing** a class of course c is not of principal importance because a completed schedule requires identification of the number of students **starting** a class of course c . The difference between the number of students starting and finishing a class of course c ,

attrition, will be considered when determining an estimate of the number of students finishing a class of a course with subsequent assignment to a class of a successor course.

D. OUTLINE

In Chapter II, gives details of MOS training in the communications and electronics fields. Chapter III presents two mathematical formulations of a MIP for the scheduling of classes at MCCES. The first MIP is a basic model and the second is a refinement of the basic model to allow for any necessary violations of constraints. Chapter IV describes the implementation of the mathematical model and Chapter V presents computational results. In the final Chapter, a discussion of the solution to the integer program is presented along with recommendations for further development and improvement.

II. PROBLEM DESCRIPTION

The problem faced here is the development of a schedule of classes for the courses MCCES provides. This problem involves establishing the number of classes of a course necessary to meet the published requirements, identifying starting dates of each class and specifying the number of students to start each class.

A. NEED FOR A SCHEDULE

MCCES must administer 53 different courses and manage approximately 4000 students to meet the Marine Corps' annual training requirements for MOSS in the communications and electronics fields. A schedule to achieve this must be developed each year to meet changing requirements.

The identification of the starting dates of all the classes of each course assists the Marines Corps assignment branch in matching the arrival of students to MCCES with the starting dates of the initial class they are to attend at MCCES. This policy is intended to minimize the time students wait for a class to start. For students having completed a class of a course and designated to attend a successor course, the schedule of classes will identify the earliest date the students may start the successor course. The amount of time the students spend waiting for a class of a successor course

to begin after completion of a predecessor course is a measure of efficiency of a schedule. A schedule of all classes of each course also provides a means to determine when students will complete training and proceed or return to duty.

B. SCHEDULE DEVELOPMENT

A completed schedule has many components, but the critical elements are the starting date of each class and number of students scheduled for each class. The other elements of the schedule, class ending date, class designation, and date students must report to MCCES (in the case of class of a course with no predecessor), can be derived from the first two elements.

The total number of students to be scheduled for training in each course for the next fiscal year is developed by Headquarters Marine Corps and is published in a Training Quota Memorandum (TQM) in December of each fiscal year. Class scheduling is currently done manually by MCCES personnel modifying the previous year's schedule, making any adjustments necessary to account for changes in the demand. This manual scheduling makes as few changes as possible to the existing schedule and thus does little to change the amount of time students spend waiting for a successor course to begin, or equivalently, change the average number of students in inventory. Once a schedule is completed by MCCES, it is forwarded to Headquarters Marine Corps for endorsement.

C. IMPROVING THE SCHEDULE

At this time, there is no measure in use for the amount of delay created by the current scheduling method. For the purpose of this thesis, delay is defined as the days a student waits to start a successor course after completing a predecessor course. Each day a Marine spends waiting for a class of a successor course to begin is one man-day of delay. An improved schedule is a schedule which minimizes total man-days of delay over the planning horizon. Improving the schedule benefits the Marine Corps by enabling Marines to return to duty at the earliest possible time after having completed all required training.

D. STRUCTURE OF THE COURSES

Due to the related material presented in many of the courses at MCCES, the courses are divided into mutually exclusive groups. Each grouping has a unique initial course and all courses within a group have a particular ordering establishing one or more sequences of courses. Many of the course groups consist of a single sequence of courses having one or more courses in the sequence. Development of a schedule to reduce waste in this type of group may be trivial in that a class of a successor course may start as soon as sufficient number of students have completed classes of the predecessor course.

A far more complicated grouping of courses is one which has as many as 17 courses, has multiple course sequences, with each sequence containing up to four courses. A network representation of this type of course group is depicted in Figure 1. The MIP presented in Chapter III will handle course groups of this level of complexity.

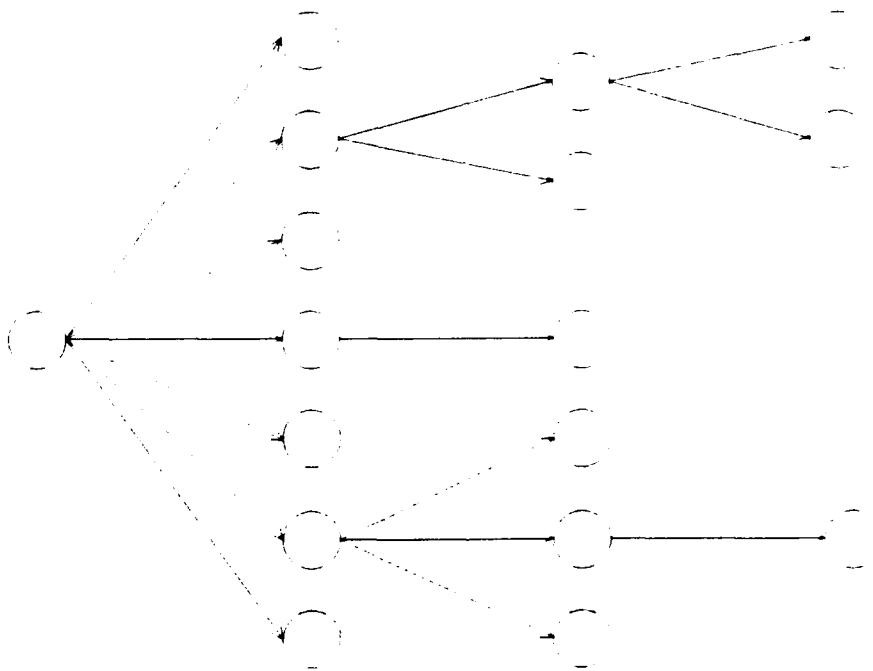


Figure 1. Directed network diagram of a course grouping beginning on the left with the unique initial course. Nodes i represent courses, arcs (i,j) indicate the course j is uniquely preceded by course i . Each echelon, left-to-right, represents a transition to successors courses from the prerequisite course.

III. MODEL DEVELOPMENT

This Chapter develops the mathematical model for scheduling and explains its features. The basic model has a generalized network flow component identifying the flow of students through a sequence of courses to meet the annual training requirement for each of the course types. However, the addition of constraints such as maximum and minimum class size complicates the model and increases the difficulty of obtaining an optimal solution.

A. BASIC MODEL

1. Indices

- c, c' - courses
- t, τ - training days
- C - set of courses in a grouping
- C^c - set of courses which are successors to course c , $C^c \subset C$
- T - $\{-235, -234, \dots, 235, 236\}$, set of training days in two consecutive fiscal years. The positive elements represent the training days of the fiscal year being planned and all other values represent training days in the immediately preceding fiscal year

2. Data

- MAX_c - upper bound on class size for course c
- MIN_c - lower bound on class size for course c
- L_c - number of training days necessary to complete a class of course c
- TIP_c - demand for course c in the planning period.
The number of students required to start training in course c during the planning period
- $CPTY_c$ - maximum number of concurrent classes of course c
- $INTV_c$ - minimum number of days between starting dates for course c
- $ATTR_c$ - fraction of students starting course c who fail to complete course c
 $0 \leq ATTR_c \leq 1$
- Y_{ct} - number of students in a class of course c beginning on day t for $(1-L_c) \leq t \leq 0$
- H_{c0} - number of students in inventory on day 0
- Z_{ct} - parameter equal to 1 if a class of course c starts on day t for $(1-L_c) \leq t \leq 0$, and equal to 0 otherwise
- A_c - cost of a student who has completed a class of course c and is in inventory

MAX_c , MIN_c , and L_c are obtained from the Course Descriptive Data (CDD), a document which is periodically reviewed and updated and remains on file with the individual course coordinators. The value of L_c is fixed and is proportional to the amount and difficulty of the material presented in each course syllabus. The type of instruction, lecture or lab, also contributes to determining course length. MAX_c is limited by size of classrooms being used, amount of equipment available for use in lab sessions and the number of instructors assigned to teach classes of course c . MIN_c identifies the minimum number of students in a class of course c which ensures resources are efficiently used. For instance, the scheduling of a class for one student requiring two instructors and a classroom wastes valuable resources. Having a student remain in inventory until MIN_c students are available to start course c is usually preferred to expending valuable instructor (and other) resources to train a small number of students.

A Training Quota Memorandum (TQM) is published once a year and specifies values of TIP_c for each course. The values identified in the TQM for TIP_c are expressed in terms of the total number of students which must be scheduled to start each course in the upcoming fiscal year.

$INTV_c$ and $CPTY_c$ must be determined by persons familiar with day-to-day operation of each course. These values are

based on limitations such as classrooms being shared by other classes of course c , the qualifications and number of instructors assigned to the course, and the type of equipment used during practical application sessions. $INTV_c$ is most closely related to the number and qualifications of instructors assigned to a course. Material presented in a course is segregated into blocks of instruction with a block usually having duration $INTV_c$. Blocks of training within a course routinely have a specific ordering which must be followed. It is possible that not all instructors assigned to teach a course are qualified to teach all blocks of a course. If only one qualified instructor is available to teach the first block of a course, and there is a class undergoing instruction in the block, no additional classes of course c may begin until that instructor is available which will occur after $INTV_c$. As another example, consider a block of training which requires the use of certain test equipment which is only available in sufficient quantity for one class. Once a class that is using the equipment has completed the block of training, another class could enter the block of training which uses the equipment.

The values of Y_{ct} are obtained from the schedule of classes for the fiscal year immediately preceding the fiscal year for which classes are being scheduled. The Y_{ct} are only defined for $(1-L_c) \leq t \leq 0$ which implies a class starting on

day t finishes in the fiscal year being planned or finishes on the day immediately prior to the first day of the fiscal year being planned. H_{c0} is the number of students who have completed course c and are in inventory on the day immediately prior to the first day of the fiscal year being planned. Z_{ct} identifies a class of course c being started on day t for $(1-L_c) \leq t \leq 0$ and equals 1 when $Y_{ct} > 0$, and equals 0 otherwise.

The cost per day of students in inventory, A_c , is specified by the user. It may be the per day salary of the typical rank of the student in inventory or any other reasonable measure of interest.

3. Decision Variables

- y_{ct} - number of students in a class of course c beginning on day t . $y_{ct} = Y_{ct}$ for $(1-L_c) \leq t \leq 0$ (this handles classes which start in the previous fiscal year but finish in the current planning period or on the last day prior to the current planning period)
- z_{ct} - binary variable equaling 1 if a class of course c starts on day t and 0 otherwise. $z_{ct} = Z_{ct}$ for $(1-L_c) \leq t \leq 0$
- h_{ct} - number of students in inventory (waiting for the next course) on day t , having completed a class of course c . $h_{c0} = H_{c0}$

4. Formulation

Stated simply, the overall objective is to get students through a sequence of courses with as few man-days of delay as possible. The lengths of the individual courses are fixed and the demand for each of the courses is also fixed so the time a student spends waiting between the end of a course he will attend and the beginning of any subsequent courses he may attend must be minimized. To achieve this objective, a function which minimizes the total man-days of inventory is used in the basic MIP model.

The mathematical description of the model is as follows:

Basic Model

$$\text{minimize } \sum_c \sum_{t \geq 1} A_c h_{ct}$$

Subject to:

$$\sum_{t \geq 1} y_{ct} = \text{TIP}_c \quad \forall c \quad (1)$$

$$y_{ct} \leq \text{MAX}_c z_{ct} \quad \forall c, t \geq 1 \quad (2)$$

$$y_{ct} \geq \text{MIN}_c z_{ct} \quad \forall c, t \geq 1 \quad (3)$$

$$\sum_{\tau = t - (\text{INTV}_c - 1)}^t z_{c\tau} \leq 1 \quad \forall c, t \geq 1 \quad (4)$$

$$\sum_{\tau = t - (L_c - 1)}^t z_{c\tau} \leq \text{CPTY}_c \quad \forall c, t \geq 1 \quad (5)$$

$$h_{c(t-1)} + (1-ATTR_c)y_{c(t-L_c)} \geq \sum_{c' \in c^c} y_{c'/t} + h_{ct} \quad \forall c, t \geq 1 \quad (6)$$

$$0 \leq y_{ct} \quad \forall c, t \geq 1$$

$$0 \leq h_{ct} \quad \forall c, t \geq 1$$

$$z_{ct} \in \{0, 1\} \quad \forall c, t \geq 1$$

$$y_{ct} \equiv Y_{ct} \quad \forall c, 1-L_c \leq t \leq 0$$

$$h_{ct} \equiv H_{ct} \quad \forall c, 1-L_c \leq t \leq 0$$

$$z_{ct} \equiv Z_{ct} \quad \forall c, 1-L_c \leq t \leq 0$$

5. Explanation of Constraints

Constraints (1) require that the total number of students scheduled to start each course over the planning period must meet the demand. Constraints (2) and (3) are variable upper bounds and variable lower bounds, respectively. These constraints specify that if a course is scheduled to begin on day t , the course must contain no fewer than MIN_c students and no more than MAX_c students.

Constraints (4) stipulate that no more than one course of a given type may begin within a specified interval. Also, the number of concurrent classes of a given course type must remain within the capacity for that course, represented by constraints (5).

Constraints (6) require that the number of students scheduled to start a successor course on day t not exceed the

number of students which have completed that course's unique predecessor. The students scheduled for a successor course may be drawn from a class of a predecessor course completed on the day prior to the current day or may be drawn from an inventory of students who have completed a class of the predecessor course prior to the previous day. It is not necessary for a predecessor course to have started in the current planning period.

Constraints (6) are generalized network flow balance constraints stated as inequalities because not all students are required to attend an additional course. Students only proceed to successor courses in accordance with a successor course's training requirement. The flow balance constraints are "generalized" because of attrition: the exact numbers of students who finish a class they start is not known but an estimate of this number is $(1 - \text{ATTR}_c)y_{ct}$. Of course, the number of students who finish a class is an integer but since ATTR_c is an estimated fraction, $(1 - \text{ATTR}_c)y_{ct}$ will also be a fraction in most cases. This causes noninteger coefficients in constraints (6) and will likely cause noninteger class sizes in the schedule. The issue of noninteger class sizes is addressed in Chapter IV.

Examination of the original formulation reveals the potential for tightening the linear programming relaxation of the model by adding constraints which are redundant in

defining the integer feasible region but remove unnecessary parts of the feasible region for the model's continuous relaxation. Let $\lceil x \rceil$ denote the smallest integer greater than or equal to x and let $\lfloor x \rfloor$ denote the largest integer less than or equal to x . The number of classes scheduled for course c must be at least $\lceil \text{TIP}_c / \text{MAX}_c \rceil$ and at most $\lfloor \text{TIP}_c / \text{MIN}_c \rfloor$ and these requirements can be explicitly added to the model:

$$\sum_{t \geq 1} z_{ct} \geq \lceil (\text{TIP}_c / \text{MAX}_c) \rceil \quad \forall c \quad (7)$$

$$\sum_{t \geq 1} z_{ct} \leq \lfloor (\text{TIP}_c / \text{MIN}_c) \rfloor \quad \forall c \quad (8)$$

B. ELASTIC MODEL

Elastic modeling allows for the penalized violation of a set of constraints and can assist in identifying constraints which otherwise would render a model infeasible in the classic sense. It is possible that a given set of data may prevent a solution to the model specified by the constraints. Constraints (1) through (8) are rigid constraints in that they do not allow for any violations to occur without rendering the model infeasible. For instance, given CPTY_c and MAX_c , it may simply be impossible to meet the demand TIP_c .

Violation of a set of constraints may be reasonable. Consider an example where course c has a minimum class size of five. If four students are presently available to start

training in this course in the rigid model they must remain in inventory until at least one additional student is available. Constraints (3) may be modified to allow four students to start course c and prevent any further accumulation of man-days in inventory. It may be preferable to "pay a penalty" and violate the minimum class size restriction rather than have students remain in inventory for an extended period of time, especially if the system has enough capacity for an additional class of course c. Except when students must work in groups of size MIN_c in order to carry out certain experiments, class size restrictions are set for convenience and slight deviations may be acceptable.

However, elastic versions of certain constraints of this model may be neither acceptable nor applicable. Constraints (1), (2), (3), (4) and (5) will be elasticized to allow penalized violations of the demand, class size maximum, class size minimum, minimum starting interval of consecutive classes and maximum capacity of concurrent classes of course c, respectively. Constraints (6) must remain rigid because material presented in successor courses requires the mastery of topics presented in predecessor courses.

Since the previous formulation of the model does not address the issues identified above, the following model is presented as a modification to the basic model.

1. Additional Indices

None.

2. Additional Data

- B_{ct} - cost of beginning a class of course c on day t
- $MAXPEN_{ct}$ - penalty per student for violating maximum class size of course c starting on day t
- $MINPEN_{ct}$ - penalty per student for violating minimum class size of course c starting on day t
- $TIPPEN_c^+$ - penalty per student for exceeding the annual training requirement of course c
- $TIPPEN_c^-$ - penalty per student for failing to meet the annual training requirement of course c
- $FREQPEN_c$ - penalty per class for violating the minimum interval for course c
- $EXTRAPEN_c$ - penalty per class for violating the maximum number of concurrent classes of course c

The data from the initial formulation, MAX_c , MIN_c , TIP_c , $INTV_c$, and $CPTY_c$ are unchanged. The parameters introduced for the reformulation must be specified by the user with values assigned by the relative importance the user places on the violation of the constraints to which the penalties apply and

relative to the inventory cost per student per day A_c . These penalties appear as coefficients of elastic variables in an objective function presented later. B_{ct} is added to tune the behavior of the model and is the cost on the class scheduling variable z_{ct} .

3. Additional Variables

- s_{ct} - slack variable identifying the number of students above MIN_c for a course starting on day t ,
 $0 \leq s_{ct} \leq MAX_c - MIN_c$
- $overmax_{ct}$ - number of students exceeding MAX_c for course c starting on day t
- $undermin_{ct}$ - number of students under MIN_c for course c starting on day t
- $overtip_c$ - number of students exceeding TIP_c for course c
- $undertip_c$ - number of students under TIP_c for course c
- $freqvio_{ct}$ - number of extra (>1) classes of course c starting in the interval $[t - (INTV_c - 1), t]$
- v_{ct} - slack variable identifying the number (< 1) of classes of course c starting in the interval $[t - (INTV_c - 1), t]$ (used in reformulation of a set of constraints)

extracl_{ct} - number of concurrent classes of course
c exceeding CPTY_c

4. Elastic Formulation

The new objective function includes the student inventory variable of the original objective function along with all of the elastic variables and their associated penalties, and the class scheduling variable and its new cost.

The mathematical description of the elastic model is as follows:

Elastic Model

$$\begin{aligned} \text{minimize } & \sum_{c \in C} \sum_{t \geq 1} A_c h_{ct} + \sum_{c \in C} \sum_{t \geq 1} B_{ct} z_{ct} + \\ & \sum_{c \in C} \sum_{t \geq 1} \text{MINPEN}_c \text{undermin}_{ct} + \sum_{c \in C} \sum_{t \geq 1} \text{MAXPEN}_c \text{overmax}_{ct} + \\ & \sum_{c \in C} \sum_{t \geq 1} \text{FREOPEN}_c \text{freqvio}_{ct} + \sum_{c \in C} \sum_{t \geq 1} \text{EXTRAPEN}_c \text{extracl}_{ct} + \\ & \sum_{c \in C} \text{TIPPEN}_c^+ \text{undertip}_c + \sum_{c \in C} \text{TIPPEN}_c^- \text{overtip}_c \end{aligned}$$

Subject to:

$$\sum_{t \geq 1} y_{ct} + \text{undertip}_c - \text{overtip}_c = \text{TIP}_c \quad \forall c \quad (\text{E1})$$

$$y_{ct} + s_{ct} + \text{undermin}_{ct} - \text{overmax}_{ct} = \text{MAX}_c z_{ct} \quad \forall c, t \geq 1 \quad (\text{E2/3})$$

$$\sum_{\tau=t-(INTV_c-1)}^t z_{c\tau} + v_{ct} - freqvio_{ct} = 1 \quad \forall c, t \geq 1 \quad (E4a)$$

$$\begin{aligned} z_{ct} - z_{c(t-INTV_c)} + v_{ct} - v_{c(t-1)} \\ - freqvio_{ct} + freqvio_{c(t-1)} = 0 \quad \forall c, t \geq 2 \end{aligned} \quad (E4b)$$

$$\sum_{\tau=t-(L_c-1)}^t z_{c\tau} - extracl_{ct} \leq CPTY_c \quad \forall c, t \geq 1 \quad (E5)$$

$$h_{c(t-1)} + (1-ATTR_c) y_{c(t-L_c)} \geq \sum_{c' \in C^c} y_{c't} + h_{ct} \quad \forall c, t \geq 1 \quad (E6)$$

$$\sum_{t \geq 1} z_{ct} \geq \lceil (TIP_c / MAX_c) \rceil \quad \forall c \quad (E7)$$

$$\sum_{t \geq 1} z_{ct} \leq \lfloor (TIP_c / MIN_c) \rfloor \quad \forall c \quad (E8)$$

$$0 \leq y_{ct} \quad \forall c, t \geq 1$$

$$0 \leq h_{ct} \quad \forall c, t \geq 1$$

$$0 \leq s_{ct} \leq MAX_c - MIN_c \quad \forall c, t \geq 1$$

$$z_{ct} \in \{0, 1\} \quad \forall c, t \geq 1$$

$$y_{ct} = Y_{ct} \quad \forall c, 1-L_c \leq t \leq 0$$

$$h_{ct} = H_{ct} \quad \forall c, 1-L_c \leq t \leq 0$$

$$z_{ct} = Z_{ct} \quad \forall c, 1-L_c \leq t \leq 0$$

5. Explanation of the Elastic Model

The new objective function includes all elastic variables and their associated penalties along with the inventory variable and its cost and the class scheduling variable and its cost. A positive value of an elastic variable should worsen the objective function value therefore all elastic penalties are positive. The cost of the class scheduling variable is actually negative which encourages classes to begin. This is necessary because the form of the elastic constraints (E2/3) might allow y_{ct} and $overmax_{ct}$ to both take on positive values with z_{ct} still equal to zero, meaning students could be assigned to a class which is not scheduled.

Constraints (E1) are a modification of constraints (1) to allow a penalized deviation from the demand for course c . Constraints (E2/3) combine the class minimum and maximum size constraints into a single elastic constraint which allows penalized violations of the minimum and maximum values. These constraints are combined by inserting a bounded slack variable, s_{ct} . This method of combining of linearly dependent constraints is described by Brown and Olson [Ref. 6]

and eliminates a block of constraints at the expense of adding an additional variable s_{ci} .

Constraints (E4a) and (E4b) are a modification of constraints (4) to allow for a penalized violation of the number of classes of course c starting in the interval $[t - (INTV_c - 1), t]$ if the number of classes in the interval is greater than one. Additionally, constraints (E4a) and (E4b) are reformulated as "pure network" constraints for reasons to be discussed in Chapter IV.

Constraints (E5) modify constraints (5) by the addition of the elastic variable $extracl_{ci}$. The inclusion of $extracl_{ci}$ allows for a penalized violation of the maximum permitted number of concurrent classes of course c .

Constraints (E6) remain unchanged from the initial formulation. They are not elastic because the predecessor-successor relationship must not be violated. A breach of these constraints would mean students could be scheduled for a successor course without having completed that course's unique predecessor.

Constraints (E7) and (E8) are identical to constraints (7) and (8) of the original model. The purpose of these constraints in the rigid model is to tighten the linear programming solution of model for a quicker solution to the MIP. The inclusion of these constraints in the elastic model accomplishes the same goal. With constraints (E7) and (E8)

remaining rigid, constraints (E2/3) are restricted to minor violations of MAX_c and MIN_c .

IV. MODEL IMPLEMENTATION

The MCCES scheduling problem represented by the elastic model of Chapter III is solved by coding the model in GAMS (General Algebraic Modeling System) and solving the model using the X-System. GAMS is a model generator and solver interface used for linear, nonlinear and integer programming problems [Ref. 7]. GAMS generates mathematical models using algebraic statements which are easily read, easily modified and readily transported from one computer to another. The X-System [Ref. 6] is a solver used to obtain solutions to a wide range of mathematical programming problems including MIPs like the MCCES scheduling model. The version of the X-System used to solve this model resides on the Amdahl 5995-700A dual processor mainframe computer at the Naval Postgraduate School.

A. PENALTY ASSIGNMENTS

Critical to the behavior of an elastic optimization model is the proper assignment of penalty values for variables in the objective function. The penalties are assigned by the user and have values relative to the importance the user places on the violation of the constraints represented by the elastic variables. Additionally, the penalties must be proportional to the costs of other variables which appear in

the objective function. For instance, a large penalty associated with the elastic variable $extracl_{it}$, along with a small value of the cost of a student in inventory, would result in placing a large number of students in inventory before a violation of $CPTY_t$ (constraints (E5)) would occur in the model. Basic decision variable costs and elastic variable penalties used here in solving the MIP are contained in Table 1 of the Appendix.

B. SPECIAL CONSIDERATIONS

Large integer programs are known to be computationally difficult to solve. For a fiscal year which has 236 training days and for a sample set of courses (17 different courses) from Communications Electronics Maintenance School (CEMS), the elastic model generates a MIP which has 13,504 constraints, 40,458 variables and 297,266 nonzero elements in the coefficient matrix. This is a relatively large integer program that takes 115 cpu seconds on the Amdahl to generate and could not be solved in a reasonable amount of time. Reducing the size of this model and capitalizing on special features of the X-System can lessen the difficulty of obtaining an optimal solution.

1. Network Constraints

An optional basis factorization in the X-System takes advantage of pure network constraints for improved efficiency [Ref. 8]. In the basic model formulation presented in

Chapter III, constraints (4) have a consecutive one's property for the variable z_{ct} and a reformulation of these constraints into pure network constraints is accomplished by a standard procedure [Ref. 9]. The procedure requires the introduction of a slack variable, z_{ct} , to convert constraints (4) from inequality to equality constraints.

$$\sum_{\tau=t-(INTV_c-1)}^t z_{c\tau} + v_{ct} = 1 \quad \forall c, t \geq 1 \quad (4')$$

The following elementary row operations are then performed on constraints (4'): (a) the new first row, $i = 1$, is the same as the original first row, (b) for $i = 2, \dots, \text{last row}$, the new row i is obtained by subtracting original row $i-1$ from original row i . When the elastic variable, freqvio_{ct} , is incorporated into the transformed constraints, constraints (E4a) and (E4b) result.

$$\sum_{\tau=t-(INTV_c-1)}^t z_{c\tau} + v_{ct} - \text{freqvio}_{ct} = 1 \quad \forall c, t=1 \quad (E4a)$$

$$z_{ct} - z_{c(t-INTV_c)} + v_{ct} - v_{c(t-1)} - \text{freqvio}_{ct} + \text{freqvio}_{c(t-1)} = 0 \quad \forall c, t \geq 2 \quad (E4b)$$

These new constraints are mathematically equivalent to constraints (4) but are pure network constraints, with each

variable having a +1 and a -1 coefficient in the combined constraints (E4a) and (E4b).

Constraints (5) also have the consecutive one's property for z_{ct} . Since constraints (E4a) and (E4b) establish a maximal set of pure network constraints over the variable z_{ct} , a reformulation of constraints (5) will not allow additional rows to be factored and is not likely to improve efficiency.

2. Planning Period Scaling

The number of time periods in the planning horizon are identified as the number of training days in a fiscal year. The length of a class scheduled in the planning horizon is designated as the number of days necessary to complete a class of course c . All the decision variables and nearly all the constraints of the model are indexed by t , training days. To reduce the number of variables and number of constraints and thereby reduce the difficulty of obtaining an optimal solution to the MIP, all parameters associated with t are divided by the number of training days in a training week.

For the MCCES scheduling problem, five training days are considered a training week and scaling of the sets indexed by t by the number of training days in a training week reduces the number of time periods in a fiscal year from 236 training days to 47 (= $\lceil 236/5 \rceil$) training weeks. References to all variables indexed by t are modified to reflect the change to

a weekly resolution. The length of a course is redefined as $\lceil L_c/5 \rceil$ and represents the number of training weeks necessary to complete a class of course c . $INTV_c$ is also scaled, $\lceil INTV_c/5 \rceil$, and redefined as the minimum number of training weeks between starting dates for course c . As a convention, reference to a training week will be taken to mean the first day of that training week.

Planning period scaling has a dramatic effect on the elastic model by reducing the number of constraints and number of variables by nearly 80%, but some loss of resolution in the optimal solution occurs too. This is most easily seen by an example. Consider a simple sequence of two courses with no students in inventory and with the first course requiring 31 training days to complete. In a model having daily resolution, a class of the first course which has 10 students and starts on training day one finishes on training day 31. In an optimal case, a class of the second course might start on training day 32 with 10 students in the class and require no students being placed in inventory. If the same two courses are scheduled by a model having weekly resolution of the time periods, a class of the first course with 10 students starting at the beginning of training week one finishes at the beginning of training week seven ($= \lceil 31/5 \rceil$). The earliest possible time a class of the second course may start is the beginning of week eight since students can only start a class

of the second course on the next increment of time after completion of the first course. This schedule also does not require students to be placed in inventory in the model because placing students in inventory may only occur on the next increment of time after completion of a first course. Examination of the schedule obtained using a weekly time period resolution reveals a hidden accumulation of 40 man-days of inventory. Since the class of the first course finishes on the first day of training week seven, training day 31, the 10 students must wait until the first day of training week eight, training day 36, which places each student in inventory for four training days: training days 32 through 35. The "slack" of a course c , slk_c , is defined as the number of days before the end of a training week the course finishes and may be computed as $5 - L_c \text{ MOD } 5$. The next section will discuss how slack and the resultant hidden inventory introduced by planning period scaling can, at least partially, be removed.

The planning period scaling procedure reduces the size of the model generated from the data set identified earlier from 13,504 to 2,731 constraints, from 36,630 to 7,348 variables and from 293,962 to 26,729 nonzero elements in the coefficient matrix. This is a significant reduction in the overall size of the model and the model generation time is reduced from 116 to eight cpu seconds.

A planning period scaling factor of five is a logical choice because a "normal" training week corresponds to the weekdays of a calendar week. Smaller scale factors could be used for greater accuracy but model size and solution times will increase. For an actual daily schedule (scale factor = 1), a solution could not be obtained using the elastic model after significant computational testing.

C. SLACK REMOVAL AND CONVERSION TO A DAILY SCHEDULE

Scaling of the time periods in the planning horizon by the number of training days in a week results in a schedule which specifies the starting dates of classes in terms of the week in which a class of a course will start. Using the convention adopted earlier, the conversion from week to day is simple: A class which starts in week t is synonymous with a class which starts on training day $5(t-1) + 1$.

For classes having starting dates rescaled to a daily resolution (t now denotes a training day), the variable z_{ct} equals 1 when a class of course c begins on **training day** t and equals 0 otherwise. (Remember that at most one class of a course can begin on a given day.) The hidden delay or slack caused by forcing classes to begin on the first day of a week is at least partially removed by the following procedure:

- For each course c , compute slk_c from the weekly schedule
- Convert the weekly schedule into a daily schedule

- For each course c , define echelon_c to be the echelon of course c where the echelon of the initial course is defined to be 1 (see Figure 1)

- For all $t = 1, \dots, 236$
 - For all courses c
 - If a class of course c begins on day t
 - Define the starting date of the class as $\text{stdt}_{c,t} = t$
 - If this is the first class of course c
 - Define $\text{stdfcl}_c = t$

- For $e = 1$ to number of echelons
 - For each course c in echelon e
 - For all courses $c' \in C^e$
 - For all training days $t = 1, \dots, 236$
 - If $z_{c',t} = 1$
 - $\text{tempsslk}_c \leftarrow \text{slk}_c$
 - While [$(\text{stdt}_{c',t} > (\text{stdfcl}_{c'} + L_{c'}))$ and
 $(\text{stdt}_{c',t} > (\text{stdfcl}_{c'} + \text{INTV}_{c'}))$ and
 $(\text{stdt}_{c',t} > 1)$ and
 $(\text{tempsslk}_c > 0)$]
 - $\text{stdt}_{c',t} \leftarrow \text{stdt}_{c',t} - 1$
 - $\text{tempsslk}_c \leftarrow \text{tempsslk}_c - 1$
 - $\text{stdfcl}_{c'} \leftarrow \text{stdt}_{c',t}$
 - $\text{slk}_{c'} \leftarrow \text{slk}_{c'} + \text{slk}_c$

Adjusting starting dates to remove the slack could cause constraint violations not already existing in the schedule. The logical comparison $(\text{stdt}_{c',t} > (\text{stdfcl}_{c'} + L_{c'}))$ ensures that $\text{CPTY}_{c'}$ is not exceeded by any amount greater than the weekly schedule. The logical comparison $(\text{stdt}_{c',t} > (\text{stdfcl}_{c'} + \text{INTV}_{c'}))$ ensures that no further violations of $\text{INTV}_{c'}$ are committed and the logical comparison $(\text{stdt}_{c',t} > 1)$ ensures that the start date of a class remains in the fiscal year being scheduled.

D. NONINTEGER CLASS SIZES

The fractional estimate of the attrition, $ATTR_c$, is likely to cause noninteger class sizes in the schedule, so a modification to the flow balance constraints (constraints (E6)) is made here,

$$h_{c(t-1)} + y_{c(t-L_c)} - NATTR_c z_{c(t-L_c)} \geq \sum_{c' \in C^c} y_{c't} + h_{ct} \quad \forall c, t \geq 1 \quad (E6')$$

where $NATTR_c$ is an integer estimate of the number of students who begin course c but fail to complete the course. Constraints (E6') require that the number of students scheduled to start a successor course on day t not exceed the number of students which have completed that course's predecessor. If MIN_c were equal to MAX_c , a reasonable value for $NATTR_c$ would be $\lfloor ATTR_c MAX_c + .5 \rfloor$. However, two potential problems arise: (a) $MAX_c > MIN_c$, which is always the case, and if MIN_c students are scheduled for a class of course c , $MIN_c - NATTR_c$ might be a pessimistic estimate of those finishing (it is positive for this data, however), and (b) $\lfloor ATTR_c MAX_c + .5 \rfloor = 0$ but $ATTR_c \neq 0$. Case (a) is not a significant problem, at least for this data, because most classes scheduled are near MAX_c in size. For case (b), when $ATTR_c$ is zero, constraints (E6') and (E6) are identical. When, due to rounding, $NATTR_c$ is zero and $ATTR_c$ is not zero (MIN_c is always greater than zero by design), no students fail to finish in the model when, in practice, there is some small attrition. A potential solution

to this problem is to increase the demand, TIP_c , for course c by $[TIP_ATTR_c]$, i.e., $TIP_c \leftarrow TIP_c + [TIP_ATTR_c]$. This procedure was not attempted as it would require some judgement from an actual scheduler working with an actual schedule to evaluate.

V. COMPUTATIONAL RESULTS

The solution to the scheduling problem faced by MCCES requires the determination of the starting date for each class of a set of courses along with specifying the number of students in each class. The MIP developed in the previous chapters is implemented to minimize the delay encountered by students while waiting for additional courses. To determine improvement made by using the MIP, a comparison is made between a class schedule developed by current scheduling methods (manual scheduling) and a class schedule developed by the MIP.

A. MANUAL SCHEDULE

To establish a reference to which comparisons may be made, an existing, manually created schedule is examined to determine the amount of delay encountered by students waiting for additional courses. Schedules developed by manual methods are not significantly different from year to year and the FY 1993 schedule of classes is analyzed. To determine the number of man-days students spent waiting for courses of the Communication Electronic Maintenance School in FY 1993, a model instance is created for daily (scale factor = 1) scheduling of classes. Then, the class size variables, y_{ct} , and the binary variables indicating the start of a class, z_{ct} ,

are fixed to the values corresponding to the schedule published for FY 1993. Using the cost and penalty parameters contained in Table 1 and the course data of Table 2, the elastic model identifies 7,819 man-days of delay in that schedule. This amount of delay sets a baseline to which any improvements to the schedule may be compared.

The manually generated FY 1993 schedule of classes violates several constraints. The minimum class size of course RDRFC (MIN_{RDRFC}) is equal to six and two classes of the course RDRFC in the manually generated schedule, on training days 2 and 122, have five students scheduled. Additionally, the minimum interval which must be maintained between starting dates of classes of course c are violated several times. $INTV_{RFC}$ and $INTV_{TSRC}$ are violated three times and $INTV_{GRRRC}$ and $INTV_{RDRFC}$ are both violated once. In light of these observations, the use of an elastic model which allows penalized violations of selected constraints is reassuring.

B. AUTOMATED SCHEDULE

To determine if the use of the elastic course scheduling model decreases the number of man-days students spend waiting for additional courses (improves the schedule), the elastic model (constraints (E6') in place of constraints (E6)) is used to generate a schedule of classes for the data of Table 2 but with the values of L_c and $INTV_c$ scaled to weekly values, ie. divided by five and rounded up if fractional. The cost and

penalty parameters of Table 1 are also used as inputs to the model.

After the elastic model determines a schedule, the starting dates of courses are rescaled to a daily resolution and are then adjusted by the procedure presented in Chapter IV to reduce the slack. Then, the class scheduling variables, z_{ci} , and class size variables, y_{ci} , are held constant and another instance of the elastic model for daily scheduling (scale factor = 1) is created and solved to determine the man-days of delay present in the schedule with slack removed. The solution to this instance of the model is obtained in a total of 374 cpu seconds. This time reflects generating and solving a weekly model (220 seconds), slack removal and conversion of start dates (insignificant), generating and solving of a daily version of the model (154 seconds). The schedule specified by these steps has 2,964 man-days delay which is a decrease of 4,855 man-days of delay from a manually developed schedule. The schedule developed by the elastic model meets all constraints except for a violation of the minimum class size for the course GRDR on training days 95 and 135. The minimum class size for GRDR is six and the number of students scheduled for the classes on training days 95 and 135 is five.

To examine the behavior of the MIP to various input parameter values, several experiments have been performed. The parameters used in these experiments are identified in

Table 3 and the results of the experiments are summarized in Table 4. These experiments indicate that student delay is sensitive to penalty values but that solution times are only slightly affected. These experiments also indicate that some "tuning" of penalties may be necessary in practice, but that such tuning is not likely to cause computational problems.

VI. CONCLUSIONS AND RECOMMENDATIONS

Comparison of the student delay in schedules constructed by manual and automated scheduling methods shows that a mixed integer programming approach can provide an improved schedule. For the set of courses examined, the amount of time students spend waiting for classes of a subsequent course can be reduced by about 4,855 man-days or over 62%. (This is about 5% of the total training man-days.) For the group of courses tested, the average delay per student is 11.7 training days (man-days delay/number students entering the initial course) under the manual schedule. For the automated schedule, the average delay per student is reduced to 4.4 training days (actual delay may be higher if NATTR_i is significantly different from the actual attrition and if many classes are at or near their minimum class size). This means that if an automated scheduling procedure is used, students attending Communications-Electronic courses could proceed to their subsequent assignments much more quickly.

An alternate way to quantify the savings provided by an integer programming approach to scheduling is by assigning a dollar value to the delay. If the average daily wage of a student who is waiting for an additional course is \$30 [Ref. 10], the schedule produced by the MIP would

save approximately \$145,650 in salary of students waiting for subsequent courses.

When the students in inventory are collected among all course groups at MCCES, the population may at times be quite large. Another saving created by reducing the number of students in inventory is the reduction in the number of supervisory personnel required to manage these students.

The use of a mixed integer programming approach to the scheduling of classes at MCCES holds the potential for a large improvement in efficiency of the training of students at MCCES. The user should be aware that $NATTR_c$ is typically a pessimistic attrition number based on class sizes being near MAX_c . If class sizes scheduled are small, it might be reasonable to adjust $NATTR_c$ downward. Reformulations of the models presented in this thesis using fractional attrition, $ATTR_c$, may produce solutions with integer class sizes and such reformulations could be explored. Additionally, the development of a user-friendly interface would be valuable.

The use of this type of scheduling model may be applicable with little or no modification to many armed forces schools. This model should apply to other schools which have multiple instances of sequential courses and which train large numbers of students each year.

APPENDIX

MODEL INPUT PARAMETERS

TABLE 1. COST AND PENALTY VALUES

Parameter	Value
A_c	0.01
B_{ca}	-0.05
$MAXPEN_{ca}$	8.0
$MINPEN_{ca}$	0.8
$TIPPEN_c^+$	8.0
$TIPPEN_c^-$	8.0
$FREQPEN_c$	80.0
$EXTRAPEN_c$	80.0

The values of parameters in this table are the values which are used as inputs to the elastic model presented in Chapter III. These values may be changed by the user to achieve different behavior from the elastic model.

TABLE 2. FY 1993 DATA FOR COMM-ELEC MAINTENANCE SCHOOL

COURSE	MAX _c	MIN _c	L _c	TIP _c	INTV _c	CPTY _c	ATTR _c
BEC	30	15	56	668	5	12	.169
RDRFC	12	6	20	49	20	1	.030
MRC	12	4	79	0	10	4	.037
TSRC	14	8	71	98	30	4	.075
TACC	12	5	83	19	83	2	.086
TDCC	9	6	98	7	98	2	0
ACEOC	6	1	63	12	63	1	0
RFC	24	12	30	359	9	4	.015
AIMS	24	6	50	31	50	1	.067
GRDR	8	6	37	10	37	1	.317
TAOM	6	3	40	12	40	1	.027
ARRC	10	4	67	23	17	3	.028
MEMC	15	10	100	30	50	2	0
GRRC	24	12	78	292	10	6	.072
ARDRA	12	4	69	14	69	1	0
ARDRB	12	4	40	17	40	1	0
PSMC	8	4	17	18	17	1	0

The data in this table is specific to the set of courses for the Communication-Electronics Maintenance School. The values are required as inputs to the elastic model of Chapter III.

TABLE 3. EXPERIMENTAL COST AND PENALTY VALUES

Parameter	Experiment 1	Experiment 2	Experiment 3	Experiment 4
A_{α}	0.01	0.01	0.01	0.01
B_{α}	-0.05	-0.05	-0.05	-0.05
$MAXPEN_{\alpha}$	8.0	4.0	2.0	1.0
$MINPEN_{\alpha}$	0.8	0.4	0.2	0.1
$TIPPEN_c^+$	8.0	4.0	2.0	1.0
$TIPPEN_c^-$	8.0	4.0	2.0	1.0
$FREQPEN_c$	80.0	40.0	20.0	10.0
$EXTRAPEN_c$	80.0	40.0	20.0	10.0

The values of the parameters of this table are variations of the parameters contained in Table 1. The values are changed to examine the response of the elastic model to different input parameter values.

TABLE 4. EXPERIMENTAL RESULTS

Experiment	Solution Time (sec)	Man-days Delay	Violations
1	374	2964	$Y_{GRDR,95} = 5$ $Y_{GRDR,135} = 5$
2	387	3416	$Y_{GRDR,135} = 4$
3	410	4688	$Y_{GRDR,60} = 4$
4	429	3248	$Y_{GRDR,70} = 3.75$ $Y_{ARDB,5} = 3$

This table displays the response of the elastic model to the differing parameter values of Table 3. In addition to the violations identified for Experiment 4, five other classes in the schedule have fractionated class sizes.

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